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91-06717/

CRT-793

Optimizing Switching Evolution in a Multi-Service Environment

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Centre de recherche sur les transports — Publication #793

August 1991

Dépôt légal – 3ème trimestre 1991

ABSTRACT

In this paper we address the problem of optimally planning the evolution of switching in a local telephone network. This problem belongs to the family of multilocation capacity expansion problems and consists in determining the choice and time of placement of switching equipments (switches and remote concentrators) at the various nodes of the network over a long-term planning horizon, while satisfying demands and minimizing costs. This optimization must be carried out in the context of multiple services and equipment types, as well as budget and other global constraints. We present a modeling approach that takes all of this into account without restricting the solutions to be single-transition strategies and yields an integer linear program that remains tractable for real-sized networks. A combined branch-and-bound/column-generation algorithm is proposed to solve it. Finally we show how this model can be integrated into a decision-support system to provide planners with a powerful and flexible planning tool.

Keywords: switching evolution, telecommunications network, multiple services, branch-and-bound, column generation.

RÉSUMÉ

Dans cet article nous abordons le problème de la planification optimale de l'évolution de la commutation dans un réseau téléphonique local. Ce problème appartient à la famille des problèmes d'expansion de capacité sur plusieurs sites et consiste à déterminer quels équipements de commutation devraient être installés aux divers noeuds du réseau et à quel instant, de façon à satisfaire les demandes tout en minimisant les coûts. Cette optimisation doit s'effectuer sur un horizon de planification de plusieures années, en considérant simultanément plusieurs services et types d'équipements, ainsi qu'en respectant des contraintes globales sur les ressources (budgets, équipements, ...). Nous présentons une approche de modélisation qui tient compte de tous ces aspects sans pour autant restreindre les solutions à des stratégies à transition unique. Celle-ci permet de formuler le problème comme un programme linéaire en nombres entiers dont la dimension demeure raisonnable pour les problèmes réels. Un algorithme de résolution combinant les approches de "branch-and-bound" et de génération de colonnes est ensuite proposé. Enfin, nous indiquons comment ce modèle pourrait être intégré en un système d'aide à la décision puissant et flexible.

Mots-clés: évolution de la commutation, réseaux de télécommunications, multi-services, branch-and-bound, génération de colonnes.

Since its introduction in telephone networks, digital technology has brought about numerous changes which have expanded the horizons of customers and companies alike. In particular the use of digital switching equipment has made technologically feasible and economically affordable the coexistence of the traditional voice service or POTS (for Plain Old Telephone Service) with many new attractive services such as holding or re-routing calls, filtering of in-coming calls, data and video transmission, and many others.

In addition to this growing appeal for the end-user, digital technology has also provided the telephone industry with a broad range of new serving arrangements like the all important remote concentrator option. Indeed, although analog remoting is technologically feasible, it is only with the advent of digital technology that the remoting option has shown its full potential and economic attractiveness. In short this option corresponds to serving a group of customers from a relatively distant switching center through the use of a particular type of equipment: a Remote Concentrator Module (RCM). These have the property of concentrating the demand of the customers they serve on a reduced number of physical links, therefore requiring less switching ressources at the host office (the office to which the RCM is linked) than if the customers were served directly through individual links.

However, this new dimension has dramatically altered the planning process for telephone networks rendering it more complex than ever. This process involves the selection of facility types (switches, transmission systems and local distribution equipment), their configuration, and the timing of their placement and subsequent

additions such that all demands for services are satisfied in a cost-effective way. An element of significant importance in this process, and the one we specifically address here, consists in determining the *minimal-cost evolution strategy for local telephone networks* (i.e. Class 5 offices, local distribution and inter-office trunking). Problems of this type belong to the family of *multi-location capacity expansion problems*. They arise in a wide variety of contexts like manufacturing and service industries, and have generally been considered to be intractable when studied in their full complexity (Luss 1982, Shapiro et al. 1973).

Early methods for tackling this problem either deliberately omitted spatial couplings between nodes and links to concentrate on timing decision (Johnston, Kettler, and Tedesco 1976), or considered a series of "snapshots" of the network thus taking spatial couplings into account but ignoring then temporal ones (Steinhorn 1973). While both of these approaches were acceptable some years ago, they no longer are. The emergence of digital technology has made it clear, through the changes we just mentionned, that an integrated approach, i.e. one where spatial and temporal interdependencies are both accounted for, is now essential for adequate planning. To this end several integrated approaches have been proposed in the last decade. A first generation of approaches, like the one proposed by Combot, Tsui, and Weihmayer (1981), considered problems with one type of service, a single type of digital switch and allowed only one transition per node over the planning horizon. Second generation approaches, like the one due to Hoang and Lau (1984 a and b), consider several digital equipments simultaneously. They thus allow a more realistic planning than

the previous, but are still limited to problems where just one service is assumed to be present over the network, and only single-transition strategies are permitted.

Planners nowadays recognize the need to take into account more than one service in the planning process, since several services do already coexist over telephone networks and, in a sense, even compete for available capacities. Moreover, while certain services might not even be present at the beginning of the planning horizon, they can be scheduled for introduction in the course of the study period and therefore have to be accounted for. To further complicate matters, the equipments available to carry a service may be numerous, each of these equipments may be more or less specific to that particular service, and eventually incompatible with other services. All of this brings to the conclusion that, in the context of multi-service planning, confining oneself to single-transition strategies is too restrictive to provide a reasonable representation of network evolution. If one wants to adequately take multi-service into account, one must consider evolution strategies where more than one transition is allowed at each node.

To illustrate this point let us consider the following simplistic example. At the start of the planning horizon, we have at some node an early type of switching equipment carrying POTS, whose capacity is almost full, and a forecasted growth in demand over the next periods such that it will be necessary to install some kind of equipment early on to handle it. In addition, we know that a second service is scheduled to be introduced at that node later in the study and that initial demand and growth for this service are expected to be low. This new service can be supported

by one of two types of equipments: equipment A, a big and costly new switch able to carry various services including POTS, or equipment B, a small and relatively inexpensive remote which will only become available with the new service. Two alternatives are then possible: 1) install equipment A when the existing switch capacity is exhausted; or 2) install first a conventional remote to handle the excess POTS demand and later on install equipment B to provide the second service. This last option is clearly a multi-transition strategy. It is at least as reasonable and possibly more efficient than its single-transition counterpart, and should therefore be considered in the optimization process as well. This is but one example of why practitionners feel the single-transition restriction should be lifted.

In this paper, we present a modeling of the problem that explicitly takes into account these two key aspects that are multi-service and multi-transition. In addition this model also allows for re-homing of remote concentrators to host switches during the planning horizon, something most previous models did not. This third generation approach is conceived more as the central piece of a Decision Support System than as a stand alone optimization model, in the sense that it allows for a fairly high level of interaction with the user, enabling him to perform extensive scenario analysis.

The paper is organized as follows. In section 1, the problem to be solved is specified. Section 2 describes the chosen modeling approach which is based on the definition of Nodal Evolution Plans and results in a natural Dantzig-Wolfe formulation for the Switching Evolution Model (SEM). The formulation of the master problem as an Integer Linear Program (ILP) and of the column-generation sub-problems as

Dynamic Programs are respectively presented in sections 3 and 4. In each case the proposed solution procedure is described. The remaining elements of the Decision Support System are characterized in section 5. Finally, the conclusion and some comments on up-coming developments will be given.

1. PROBLEM DEFINITION

Switching Evolution as Network Evolution

Telecommunications networks have two main functions: transmission and switching of signals. Optimizing the evolution of such a network should thus normally address both the switching and the transmission evolutions. However, in the case of telephone networks these two elements do not have the same weight with regards to their economic impact. Studies made in the late 70's have shown that transmission (i.e. trunking and interfacing) amounts for only 10 to 15% of total network evolution costs (Combot and Epstein 1979). This clear dominance of switching equipment expenditures, coupled with the fact that considering trunking and switching together tremendously complicates the problem (see Helme 1986), has brought many model-builders to concentrate on switching, independently of trunking, working under the assumption that trunking costs and switching decisions are mostly independent. Works by Combot and Epstein (1979), Combot, Tsui, and Weihmayer (1981) and by Hoang and Lau (1984 a, b) illustrate this approach. The justification of this independency assumption has recently been strenghtened by an economical analysis of the industry due to Curien and Gensollen (1987). This analysis made in the context of deregulation suggests that the recent technical evolution due to digital technology will result in a

progressive separation of the transmission and switching functions of telecommunications networks. Optimizing the evolution of the whole network is thus more and more equivalent to optimizing the evolution of switching and transmission independently.

Accordingly, we have retained this assumption and used it to guide our modeling of the problem. Therefore when we talk about optimizing network evolution we mean optimizing the switching evolution. In our approach, as is the case in many others, the transmission evolution optimization is to be performed in a subsequent stage, once the optimal switch evolution strategy has been determined. This complementary problem has received a lot of attention in the litterature (see, for instance, Gavish et al. 1989 and, for a comprehensive survey, Minoux 1988) and will not be addressed here.

Characteristics of the problem

Let us now formally define the main elements of the problem and introduce at the same time some necessary terminology and notation. The network can be characterized by a set of fully connected nodes (Class 5 switching centers) $N = \{1, 2, ..., n\}$ and a distance matrix $L = [\ell_{ij}]$. This topology is assumed to be stationnary. The study time horizon is finite, of T years, and is marked by the time scale $K = \{1, 2, ..., T\}$. S is the set of services to be considered in the study. Service demands are specified at every node i, for every period k, and for each service s, through a demand matrix $D = [d_{iks}]$. Demands can be expressed in "lines", "traffic" or some other unit, the only restriction being that this measure be consistent for all services and equipments. E is the set of equipment types from which provisioning is to occur and $E = C \cup R$, where C is the set of switch types available and R the set

of remote concentrator module types. Each type of equipment can be characterized by: installation, retrieval, operating and modification costs; the set of services it can support (called set of compatible services); and the maximum serving capacity for each compatible service (expressed in the same units as the service demands). It must be pointed out that modern switches and RCMs are often modular in nature; they can thus be configured in a variety of ways, both from the hardware and software standpoints, with widely different capabalities as a result. Within the context of our model, variants of a given equipment, even though they might bear the same name, must be treated as distinct equipment types to account for these differences in their capabilities. This also explains why we consider modification costs for equipments, since a more powerful device supporting, for instance, a wider range of services may often be obtained by the simple addition of hardware modules and/or software to an already existing equipment. Such modifications can obviously be performed at a much lower cost than the outright replacement of existing equipments by new ones.

An essential characteristic of the problem is that for most equipments their serving capacities are not additive: the total serving capacity of an equipment is generally less than the sum over the set of compatible services of all its individual service capacities. This translates into the necessity of having "global" capacity constraints for every possible subset of compatible services.

A last characteristic applies to remote concentrators alone: since they do not perform "commutation" of signals, they need to be "homed" (i.e. connected) to a digital switch at all times. One will therefore need to know for every type of

RCM the number of "lines" (i.e. pairs of wires), or other chosen capacity measure, required to connect them to a digital exchange. Naturally these host-switches must be "compatible" with the RCMs that are homed to them, technology and service-wise, and they must also have enough serving capacity to accomodate these extra demands. The collection of all these homing links (for every RCM present and every planning period) constitutes what is called the *homing pattern*. They represent the last remaining element of "transmission" still present in the Switching Evolution Problem that because of the RCM particular nature has to be kept in, for otherwise provisioning RCMs would be meaningless.

Provisioning of equipments is done discretely along K and all transitions are considered to take place at the beginning of a period. Possible transitions consist in installing or removing some type of equipment, in modifying the type of some equipment already installed, or in changing a homing decision. The problem we wish to solve can then be stated as follows.

Given all of the above plus:

- the initial state of the network,
- cost and financing parameters, and
- some constraints on the availability of resources (budget and equipment);

we want to find a *switching evolution strategy* that minimizes, over the whole planning horizon, the total cost of evolution while satisfying all demands. The total cost of evolution could be computed using any one of the usual economic indicators like,

among others, the Net Present Value (NPV) or the Present Worth of Annual Charges (PWAC).

Such a strategy can be defined as:

- the type and number of equipments installed at each node as a function of time (installation and, if necessary, retrieval periods); and
- the associated homing pattern (i.e. the definition for every remote concentrator of the host digital switch to which it is connected), also as a function of time.

Equipment vs line provisioning

We feel it is important to point out that our work differs from that of others in a fundamental way: here the problem is addressed from an *equipment* rather than from a *line* provisioning point of view. The latter way of dealing with the modernization problem is the one that has been privileged by most authors in recent years, as evidenced by the works of Mason, Girard and Gu (1990), Antunes, Climaco and Craveirinha (1990), and Mason (1984). With this line provisioning approach, capacity expansion or modernization is modeled as a continuous decision process without reference to the underlying equipment support (switches and RCMs). The problem can then be formulated as a Linear Program, and a score of well known and proven solution techniques can be used to tackle it. However, it is our belief that this approach overlooks some essential aspects of the problem like, among others: the dependencies that exist between the equipments present at a given office and the service capabilities of that particular office; the impact that placing RCMs in the

network has on costs, on available capacities at host offices, and on the evolution strategy as a whole. These elements are all integer by nature and can only be addressed if equipment provisioning decisions are considered explicitely, along with the line-allocation decisions. This evidently induces mathematical formulations and solution techniques far less attractive than the ones obtained with the line provisioning approach. Nevertheless, we believe it is important that the integer programming dimension of the problem be retained as much as possible in its modeling, so that the best possible planning be made.

2. THE MODELING APPROACH

The straightforward formulation of the Switching Evolution Problem as a mathematical program proves impractical even under strongly simplifying assumptions. For instance, if one supposes the presence of at most one equipment per type, then using binary variables $x(i,k,t) = 1(=0)$ to represent the presence (absence) of equipment t at node i during planning period k and similar homing variables $y(i,j,k,r)$ (homing of remote r placed at node i to node j during period k) yields an extremely large-sized ILP with no apparent special structure. Indeed, a real-life size problem could have around $n = 50$ nodes, $T = 10$ periods, 15 types of equipments (5 switches and 10 concentrators), and 5 different services. With these values one can expect to have: $nK|E| = 50 \times 10 \times 15 = 7,500$ x-variables and $n(n-1)T|C| = 122,500$ y-variables. Counting one capacity constraint for each subset of services at each node for each period one would have $nT\left(2^{|S|-1}\right) = 15,500$ capacity constraints, and finally, for the homing pattern to be feasible, one must include one constraint to guarantee that

every concentrator has a host in every period, and another one to ensure that this host is unique, which gives $2n(n-1)T|C| = 490,000$ constraints (for a more detailed discussion refer to Soriano, 1987). This is obviously not amenable to exact or even approximate solution techniques.

A deeper analysis of the problem, shows that if it were not for the homing and resource constraints, it would separate into a series of independent nodal evolution problems. This suggests the following formulation: a set partitioning problem with additional constraints where the binary decision variables correspond to the choice of a set of Nodal Evolution Plans (or NEPs) to "cover" all nodes. These plans include all provisioning decisions needed to completely satisfy local demands for every service either by on-site switching or via remoting. All of the equipments selected in defining a NEP are considered to be installed at their full capacity. Any residual switching capacity left over after loading local demands is thus available to accomodate homing demands due to remoting at other nodes. In other words this constitutes the "hosting capacity" of that particular NEP.

This way of approaching the problem presents a big advantage which is that decisions pertaining to individual equipments are no longer explicitly part of the optimization process but are rather addressed while defining each plan; this tremendously reduces the number of constraints that need to be considered in the optimization. But this approach also creates a whole new set of problems which are related to the way that homing is handled. If a NEP includes remote concentrators, there exists a large number of ways that these can be homed to potential host-switches at other

nodes. If re-homing of remotes is allowed (which seems to be a perfectly reasonable assumption over a long planning horizon), the number of possible homing patterns for a given NEP may become astronomical. Furthermore, the hosting capacity of a NEP is not well-defined. This stems from the fact that the exact capacity available for hosting remotes is not only dependent on the equipments that are installed, but also on the specific "loading pattern" which is implemented, i.e. on how the various service demands are assigned to the equipments. As we shall see later on in section 5, if there are several versatile equipments at a node, the number of feasible loading patterns may be indeed quite large.

For these reasons, it is obviously not possible to account directly for the homing of remotes to switches in a solution approach which is based on the concept of NEPs.

At this point, it should be realized that, given the current state of algorithmic development, the level of combinatorial interaction between provisioning, homing and loading decisions is such that any single optimization model encompassing all three dimensions of the problem will prove intractable, even for moderately-sized networks. Some decoupling of these dimensions is thus required, if the problem is to be solved at all. The approach which we propose consists in first solving the *equipment provisioning problem*, using aggregate measures to account for the loading and homing aspects. This is done by the Switching Evolution Model (SEM) which yields a Switching Evolution Plan (SEP) for the whole network. Once equipments have been selected, a Homing and Loading Procedure (HLP) can then be used to ascertain that there exists a feasible homing and loading pattern for the chosen SEP.

If no such pattern can be found, one goes through a "Homing Adjustment Loop" to modify some of the parameters of the SEM, which is then solved again. This procedure is illustrated in Figure 1.

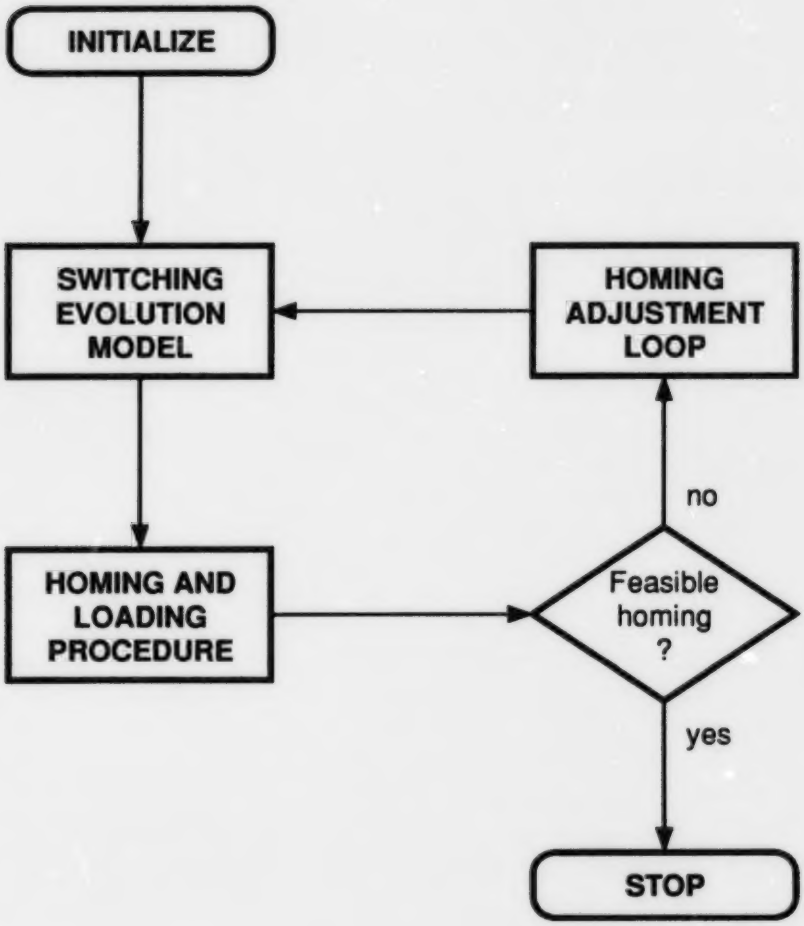

Figure 1: The overall solution procedure

Given this two-step procedure, it now becomes attractive to solve the SEM as a set partitioning problem with additionnal constraints. Some of these constraints are straightforward and account for limitations on the available resources. The remaining constraints handle the homing of remotes to switches in an aggregate fashion. The basic idea behind these constraints is that a SEP will be feasible only if, at each time period and for each service (or group of services), enough hosting capacity is available *in the network* to handle the homing demand (again computed for the whole network). Since, as was stated earlier, the exact hosting capacity or homing demand of a NEP cannot be specified exactly without loading the equipments, we rely on estimates of these capacities and demands which are determined for every NEP using simple computations (see Section 4). These estimates provide useful information on the impact of any given NEP on the overall solution and make it possible to discriminate between plans not only through local criteria (i.e. the NEP costs), but also on the basis of global considerations (through a pricing of hosting capacities and homing demands).

This formulation of the SEM allows for a simple implementation of the Homing Adjustment Loop: if during the execution of the Homing and Loading Procedure, it is found that there is a lack of hosting capacity to home some remotes, the right-hand side parameters of the corresponding homing constraints of the SEM are increased by a small amount, so that eventually the SEM will produce a HLP-feasible solution.

Before proceeding to a detailed description of the SEM itself, it should be stressed that in the proposed approach, there is a single optimization model: the SEM which

provides us with the solution which we are looking for. The Homing and Loading Procedure and the Homing Adjustment Loop are simply auxiliary devices which are implemented to guarantee a feasible SEP.

3. THE SWITCHING EVOLUTION MODEL

3.1 Mathematical model

As we mentioned in the previous section, the Switching Evolution Model (or SEM) is based on a set partitioning formulation. Let the subscript p identify a particular Nodal Evolution Plan (NEP). Recalling that only feasible NEPs are considered (i.e. NEPs in which all local demands are satisfied through switching or remoting), the set of all feasible plans for node i is $P(i)$. To every NEP p, $p \epsilon P(i)$, is associated a nodal evolution cost c_{ip} and a decision variable x_{ip}, with

$$x_{ip} = \begin{cases} 1, & \text{if plan } p \text{ is chosen for node } i; \\ 0, & \text{otherwise.} \end{cases}$$

We have to select one and only one plan for each node in the network so the following multiple-choice constraint must be satisfied:

$$\sum_{p \epsilon P(i)} x_{ip} = 1, \qquad \forall\, i \epsilon N. \tag{1}$$

To account for possible limitations in the availability of some resources (i.e. budgets and equipments) we use the following set of constraints:

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \alpha_{ipk}\, x_{ip} \leq b_k\,, \qquad \forall\, k \epsilon K, \tag{2}$$

where α_{ipk} is the vector of resource requirements of plan p at node i for period k, and b_k is the vector of resources available in period k.

With each plan p are associated *homing demands* per period and per service denoted by $\delta_{ipk}(s)$ and *residual switching capacities* identified by $\varphi_{ipk}(s)$ (taking nonnegative values). Recalling that equipments are versatile and that their capacities are global mesures affecting subsets of services, we can write the aggregate homing constraints we mentionned in section 2 as follows:

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \varphi_{ipk}(Q) x_{ip} - \sum_{s \epsilon Q} y_{ks} \geq \epsilon_k(Q), \qquad \forall k \epsilon K, \forall Q \epsilon \mathcal{P}(S) \tag{3}$$

$$-\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \delta_{ipk}(Q)\, x_{ip} + \sum_{s \epsilon Q} y_{ks} = 0, \qquad \forall k \epsilon K, \forall Q \epsilon \mathcal{P}\,(S) \tag{4}$$

where the y_{ks} variables represent the volume of service s that is homed during period k (with $y_{ks} \geq 0, \quad \forall k \epsilon K, \ \forall s \epsilon S$) and $\mathcal{P}(S)$ is the collection of all subsets of S (i.e. $\mathcal{P}(S) = \{Q \mid Q \subset S\}$). Constraints (4) ensure that all homing demands generated by the chosen NEPs will effectively be "homed" to some other node for switching, while constraints (3) guarantee that there is globally enough residual switching capacity available over the network to "host" all homed demands. The homing variables y_{ks} act as driving belts between (3) and (4) ensuring thus the "feasibility" (in a loose sense) of the solution proposed by the SEM.

The $\epsilon_k(Q)$ in constraints (3) are the adjustment parameters through which the Homing Adjustment Loop acts on the SEM. These are input parameters that are initially set to zero and then increased iteratively if the solution provided by the

SEM does not allow the HLP to find a strictly feasible homing and loading solution. Increasing any $\epsilon_k(Q)$ between two iterations in the main loop of the model will force the SEM to select a new solution providing more hosting capacity for that service subset Q than the previous one. Note that we need only to play on constraints (3), because they alone act on the hosting capacity of the strategy to be chosen.

Finally, if we denote by c'_{ks} the average unit cost of homing for service s in period k, the optimization model obtained consists in minimizing the following objective function

$$z = \sum_{i \epsilon N} \sum_{p \epsilon P(i)} c_{ip}\, x_{ip} + \sum_{k \epsilon K} \sum_{s \epsilon S} c'_{ks}\, y_{ks} \qquad (5)$$

subject to constraints (1) to (4).

3.2 Relaxed model

The number of constraints in the SEM is proportional to the size of $\mathcal{P}(S)$. Now, recalling from section 2 that the problems to be solved will have in the order of 50 nodes, 10 decision periods, 5 services and some 15 types of equipments, we will therefore have $|\mathcal{P}(\mathcal{S})| = 2^5 - 1 = 31$ different subsets Q. The SEM will thus count 50 constraints (1), 2 x 10 x 31 = 620 constraints (3) and (4), plus the (2) constraints (not too numerous in general), giving a total of approximately 700 constraints. Solving problems of this size does not constitute an overwhelming task in se, but since this problem is part of an iterative loop, solving it a good number of times in sequence might prove rather time consuming, even more so when one considers the fact that the number of decision variables is huge.

However, it is quite clear that a good number of the aggregate homing constraints (3) and (4) can be expected to be inactive since they are written for every subset of S. Now, noting that many of the services considered have similar characteristics (i.e. same set of compatible equipments, comparable consumption of resources, ...), we can accordingly regroup them in some way. To do so we introduce the concept of *service classes* and define a "basic service" class (i.e. essentially POTS) and one or more "special services" classes as deemed necessary by the context. This service aggregation translates itself in the SEM formulation by retaining only those homing constraints where Q is either an individual service, a subset of S corresponding to one of the identified service classes, or the whole service set S. Note that, if we consider a basic service class comprising only POTS, there is no need to include homing constraints for it, since they are already present in the individual service constraints. Considering that this is the case and that there is only one class of special services, then the number of homing constraints needed in this relaxed formulation, for each period, is: 2 for each individual service, 2 for the special services class, and 2 global constraints; which gives a total of ((2x5) + 2 + 2) x 10 = 140 constraints using the same parameter values as before. Each additional service class introduces 20 more constraints, but definition of only a few classes should be sufficient. This relaxed formulation should thus have around 200 to 250 constraints for real-life sized problems, which is a far cry from the 700 we had with the complete formulation.

Recall from the definition of the NEPs that all equipments are considered to be installed at their full capacity. Since any unused *hosting capacity* constitutes an

unwarranted cost (in real life extra capacity would generally not be installed until needed), we have to provide a means to discount the cost of the unused portion of the installed hosting capacity from the total cost of evolution strategies. This is done by taking explicitly into account the slack variables on the *total residual capacity* for each period (i.e. constraints (3) where $Q = S$, noted u_k, and associating to them prices $c_k^{''}$ representing the average (across equipment types) marginal cost of a unit of hosting capacity in period k. These $c_k^{''}$ act as a kick-back on the superfluous hosting capacity.

Considering the previous example (i.e. one special service class and the basic service class contains only POTS), the relaxed formulation of the SEM can be stated as follows:

SEM (R)

$$\text{Minimize } Z = \sum_{i \epsilon N} \sum_{p \epsilon P(i)} c_{ip}\, x_{ip} + \sum_{k \epsilon K} \sum_{s \epsilon S} c_{ks}^{'}\, y_{ks} - \sum_{k \epsilon K} c_k^{''}\, u_k \quad (6)$$

subject to

$$\sum_{p \epsilon P(i)} x_{ip} = 1 \qquad ,\forall i \epsilon N; \quad (7)$$

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \alpha_{ipk}\, x_{ip} \leq b_k \qquad ,\forall k \epsilon K; \quad (8)$$

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \varphi_{ipk}\,(s) x_{ip} - y_{ks} \geq \epsilon_k(s) \qquad ,\forall k \epsilon S, \forall s \epsilon S; \quad (9)$$

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \varphi_{ipk}(S')\, x_{ip} - \sum_{s \epsilon S'} y_{ks} \geq \epsilon_k(S') \qquad ,\forall k \epsilon K; \quad (10)$$

$$\sum_{i \epsilon N} \sum_{p \epsilon P(i)} \varphi_{ipk}(S)\, x_{ip} - \sum_{s \epsilon S} y_{ks} - u_k = \epsilon_k(S) \qquad ,\forall k \epsilon K; \quad (11)$$

$$-\sum_{i\epsilon N}\sum_{p\epsilon P(i)}\delta_{ipk}(s)\,x_{ip}+y_{ks}=0 \qquad ,\forall k\epsilon K, s\epsilon S; \tag{12}$$

$$-\sum_{i\epsilon N}\sum_{p\epsilon P(i)}\delta_{ipk}(S')\,x_{ip}+\sum_{s\epsilon S'}y_{ks}=0 \qquad ,\forall k\epsilon K; \tag{13}$$

$$-\sum_{i\epsilon N}\sum_{p\epsilon P(i)}\delta_{ipk}(S)\,x_{ip}+\sum_{s\epsilon S}y_{ks}=0 \qquad ,\forall k\epsilon K; \tag{14}$$

$$\begin{aligned} &y_{ks}\geq 0 && ,\forall k\epsilon K, \forall s\epsilon S;\\ &u_k\geq 0 && ,\forall k\epsilon K;\\ &x_{ip}\epsilon\{0,1\} && ,\forall i\epsilon N, \forall p\epsilon P(i); \end{aligned}$$

where S' is the subset of S corresponding to the special services class.

3.3 Solution procedure

The mathematical model which we just describe (SEM(R)) is a large mixed integer program. The classical approach to solve such problems is the well-known *branch-and-bound* procedure. In this case however, we have to account for an additional difficulty: for any reasonably sized problem, the number of feasible NEPs for the nodes is so large that it is not possible to enumerate them a priori. To overcome this difficulty, we propose resorting to a *column-generation* scheme which is often used in similar contexts when the number of decision variables to be considered is too large. The resulting algorithm will thus be a combined branch-and-bound and column-generation procedure.

Before describing in more detail this algorithm, we first recall briefly the basic principles of column-generation for continuous linear programs. Column-generation,

for a continuous problem, entails working with a Restricted Master Problem (RMP) in which only a subset of the decision variables first appear, and adding to it, as the algorithm progresses, new variables (columns) which have negative reduced costs in order to improve the solution. These new columns are identified by solving a column-generation subproblem (SP) which incorporates the dual information provided by the current optimal solution of the RMP. The procedure terminates with the optimal solution when there are no more negative reduced cost variables.

In our context, the column-generation subproblem has a natural interpretation, since it involves generating NEPs. In fact, there is not a single column-generation subproblem, but rather n of them, each corresponding to a specific node of the network (see section 4 for a detailed description of the subproblems), which implies that this column-generation approach is indeed Dantzig-Wolfe decomposition (see Goldfarb and Todd 1989).

Going back now to our proposed combined branch-and-bound column generation algorithm, we must first notice that although each of these procedures is rather straightforward to implement in isolation, interfacing them requires careful consideration. This systems from the fact that the standard branching rules which are used in branch-and-bound (branching on variables) are meaningless in the context of the column-generation subproblems. Should we try to use such rules, we would most probably end up with the same NEPs being generated again and again (with different variable names) in the SPs after having been excluded (by setting a specific x_{ip} variable to 0) earlier in the B&B tree. We thus need to devise more refined branching

rules that can be implemented in the SPs and still translate easily in terms of the x_{ip} variables for the RMP.

When the solution provided by the RMP is not integer, it is because for a given node more than one plan has been retained (with a fractional value for each of the corresponding $x'_{ip}s$). Naturally these plans are different one from the other, but these differences can be classified according to the element of the plan on which they occur. Indeed, it is fairly easy to see that plans for a given node can differ in only three ways:

1. on the *type* of equipments used,
2. on the *number* of equipments of a given type (when they use the same types overall), or
3. on the *existence sequence* of the equipments used (when all equipment decisions are identical, but some timing decisions differ).

Furthermore, there exists a kind of natural hierarchy between these differences since a dissimilarity on the type of equipments used by two plans can be considered as being "stronger" than one between two plans that use the same type and number of equipments, but diverge only on introduction or retrieval dates.

Therefore, defining the branching rules in the B&B algorithm according to these criteria and implementing them in a branching process that reflects their hierarchy will allow the required information to flow back and forth between the B&B and the column-generation SPs. The overall process will thus be consistent since the branching rules will have a clear meaning in both contexts. Implementing them in the RMP will amount to leaving free or fixing to zero all concerned x_{ip} variables.

In the case of the SPs, the rules will translate into forced inclusion (or exclusion) of some type of equipment or number of equipments of a particular type or, in the case of the third criterion, in the forced presence (or absence) of some equipment at a particular planning period. As shall be seen in Section 4, these can all be easily implemented into the SPs.

To recapitulate, the proposed solution approach, is based on a classical B&B algorithm that incorporates the branching process just described. The continuous relaxation of SEM(R) that provides the bounds is solved using a column-generation approach. The resulting RMP is itself solved using the simplex algorithm for linear programs; as for the column-generation SPs, their solution procedure is somewhat more complex and will be discussed in section 4. Figure 2 illustrates the structure of the overall solution approach for the SEM.

4. COLUMN-GENERATION SUBPROBLEMS

4.1 Dynamic Programming Formulation

The column-generation subproblems (SPs) we introduced can be formulated very naturally as Dynamic Programming (DP) problems for each node, where:

1. the *stages* of the decision process correspond to the periods of the planning horizon;
2. the *state* of the "system" is defined by the equipments present (at the node) at that stage (i.e., the "technological state" of the node at that time); and
3. the decisions to be taken correspond to the *transitions* that can be made from one state in a given stage (period) to some state in the next stage.

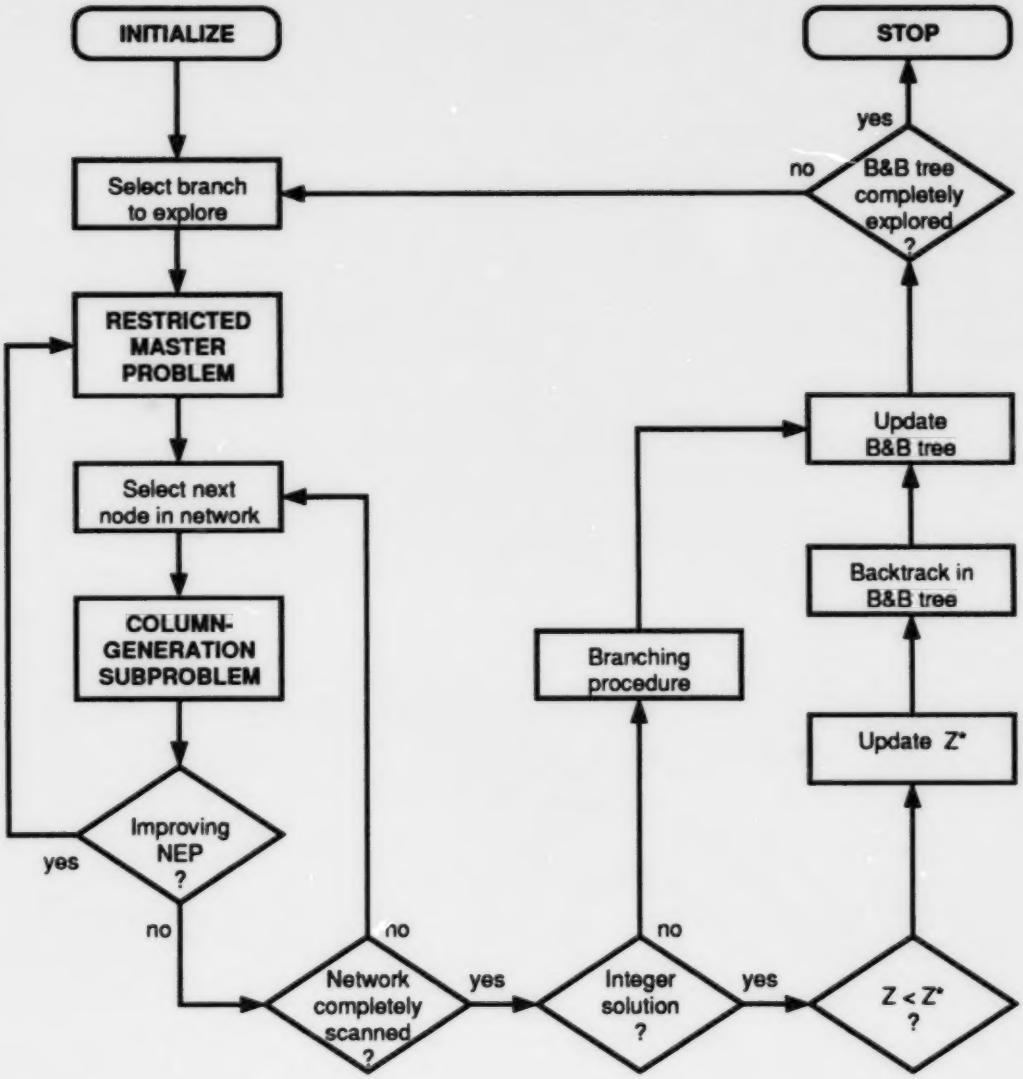

Figure 2: The combined branch-and-bound/column-generation algorithm for solving the SEM

We remind that only "feasible" plans are considered in the SEM, consequently only states that are feasible in the same sense, that is states in which the equipments present are able to completely satisfy local demands, will be considered in the DPs. Omitting node subscripts which are implicitly defined in this context, we have at any given stage k, for every state e a *local cost* $C_k^{\ell}(e)$ (i.e., operating costs of the equipments present for that period) and for every transition from state e in stage $(k-1)$ to state e' in stage k a *transition cost* $C_k^{t}(e', e)$ (i.e., cost of retrieval, installation and/or modification of any equipment that reflects the technological change undergone by the node when passing from state e' to state e at that time).

Now, let $F_k^*(e)$ be the partial objective function for being in state e at stage k (i.e. the objective function restricted to period 1 to k given that if an optimal "trajectory" has been followed from 1 to k), the *functional equation* of the process can then be written as:

$$F_k^*(e) = C_k^{\ell}(e) + \min_{e' \epsilon E_{k-1}} \left\{ F_{k-1}^*(e) + C_k^{t}(e', e) \right\} \qquad \forall e \epsilon E_k, k = 1, 2, ..., T,$$

where E_k is the set of all "feasible" states for stage k and

the boundary conditions of the process are

$$F_o^*(e) = \begin{cases} 0, & \text{if } e \text{ is the state of the node at the} \\ & \text{beginning of the planning horizon,} \\ +\infty, & \text{otherwise.} \end{cases}$$

Finally, denoting by $V(e)$ the (salvage) value of being in state e at the end of the planning horizon, we want to find the sequence of states (and thus of transitions) that will lead us to state e^* in stage T such that:

$$F^*(e^*) = \min_{e \epsilon E_T} \left\{ F_T^*(e) - V(e) \right\}.$$

The sequence of transitions leading to e^* in stage T defines the minimal cost plan for that node.

Computing the solution to these DPs can be done through the *reaching* technique (for details, see Denardo 1982) and does not present any difficulties unless the size of the DP problem is too large, or more simply stated unless the number of stages times that of states (per stage) is huge. As we indicated earlier, we expect to have about 10 stages (if T = 10 periods) which is fine, but the number of possible states can be very large, if all possible feasible states are allowed (i.e. if one considers all combinations of types and number of equipments). To prevent this, we will use a heuristic procedure to select a set of potential equipments from which the plan will be constructed. Note that this set may eventually contain several equipments of the same type, something which is perfectly acceptable, since a plan might need more than one equipment of a given type. In the Dynamic Programming problem for that node, the only states considered will be those derived from this "potential set" of equipments. By controlling the size of this set, we will thus maintain the size of the resulting PDs to a manageable level.

To describe it briefly, this heuristic selection procedure is based on a two phase scheme: satisfying first what could be called the non-service constraints (i.e. constraints originating from the branching process of the B&B or from other modules of the DSS – see section 5) and then satisfying the local service demands for each period of the horizon. The resulting set of potential equipments will therefore strictly satisfy all branching and other hard constraints (e.g., it will not contain any equipment

of an excluded type,...), and at least one state derived from it (by declaring present all or some of its elements) is capable of serving all local demands for every period. The complete description of this procedure can be found in Idrissi-Kaïtouni 1989 (pp. 39-62).

4.2 Cost parameters, capacity and demand estimates

As previously mentionned, these subproblems arise in the context of a column-generation scheme, hence the impact that the chosen NEP will have on the remainder of the Switching Evolution Plan must be an integral part of the selection criteria. Therefore, the desired solution here is not the minimum cost plan but rather the one with the smallest *reduced cost.* This is obtained by appropriately incorporating to the cost parameters used in the DPs the information provided by the dual variables associated with the RMP solution at hand.

More precisely, the transition costs described in 4.1 will be unaffected, but the local costs have to be redefined as follows. The local cost of being in state e in stage k will now be:

$$\overline{C}_k^{\ell}(e) = C_k^{\ell}(e) - A_k(e),$$

where $C_k^{\ell}(e)$ is the original "local cost" as described in 4.1, that is the total operating cost associated with state e evaluated for period k, and $A_k(e)$ is the "attractiveness" associated with being in state e at stage k, with respect to the current SEP.

This attractiveness measures the value or cost that should be discounted or added to the direct costs associated with (e, k) in order to account for the amount

of global resources it will generate or consume. There are three type of resources in our problem each identified by a set of constraints, therefore $A_k(e)$ will comprise three terms each relating to a particular resource type. Now, letting $\pi_k, \mu_k(s), \mu_k(S'), \mu_k(S), \lambda_k(s), \lambda(S')$ and $\lambda_k(S)$ be the dual variables associated with constraints (8) to (14) of SEM(R) respectively, and using letters φ, δ, and α in the same way as when formulating the SEM so that: $\varphi_k(e,s)$ denotes the estimate of the hosting capacity for service s when in state e at stage $k, \delta_k(e,S')$ identifies the estimate of the homing demand of all services $s \epsilon S'$ when in state e at stage k and so on; then $A_k(e)$ is given by the following expression:

$$\begin{aligned} A_k(e) = & \left[\sum_{s \epsilon S} \varphi_k(e,s)\mu_k(s) + \varphi_k(e,S')\mu_k(S') + \varphi_k(e,S)\mu_k(S)\right] \\ & + \left[\sum_{s \epsilon S} \delta_k(e,s)\lambda_k(s) + \delta_k(e,S')\lambda_k(S') + \delta_k(e,S)\lambda_k(S)\right] \\ & -\alpha_k(e)\pi_k. \end{aligned}$$

The first bracketed term in $A_k(e)$ relates to constraints (9) to (11) in SEM(R) and represents the value of the hosting potential provided by (e,k). The second one corresponds to the cost associated with the homing demands generated by (e,k) and relates to constraint (12) to (14). Finally, $[\alpha_k(e)\pi_k]$ accounts for the consumption by (e,k) of those resources that are limited by (8) in SEM(R).

Moreover, computing this attractiveness demands that the estimates on which it is based be themselves defined. To do so we first need to introduce some additional notation. Let $J(e)$ be the set of equipments defining state e and subscripts c and r identify those equipments of $J(e)$ that are respectively switches and remotes. Let also

$g_c(s)$ denote the serving capacity of equipment $c \epsilon J(e)$ for service $s \epsilon S$ and finally, to simplify the formulae, we will use $[A]^+$ in place of Max $\{A, 0\}$.

Then the homing demand estimates for state-period combination (e, k) will be obtained by performing the following computations in the order in which they appear:

for every $s \epsilon S$,

$$\delta_k(e, s) = \left[d_k(s) - \sum_{c \epsilon J(e)} g_c(s) \right]^+ ;$$

for the special services class S',

$$\delta_k\left(e, S'\right) = \max \left\{ \left[\sum_{s \epsilon S} d_k(s) - \sum_{c \epsilon J(e)} g_c(S') \right]^+ , \sum_{s \epsilon S'} \delta_k(e, s) \right\};$$

and finally,

$$\delta_k(e, S) = \max \left\{ \left[\sum_{s \epsilon S} d_k(s) - \sum_{c \epsilon J(e)} g_c(S) \right]^+ , \left(\delta_k(e, S') + \sum_{s \epsilon S \setminus S'} \delta_k(e, s) \right) \right\}.$$

Similarily, the hosting capacity estimates for (e, k) will be obtained by computing sequentially the following expressions:

$$\varphi_k(e, S) = \sum_{c \epsilon J(e)} g_c(S) + \delta_k(e, S) - \sum_{s \epsilon S} d_k(s);$$

for the special services class S',

$$\varphi_k(e, S') = \min \left\{ \left[\sum_{c \epsilon J(e)} g_c(S') + \delta_k(e, S') - \sum_{s \epsilon S'} d_k(s) \right]^+ , \delta_k(e, S) \right\};$$

for every service $s \epsilon S'$,

$$\varphi_k(e, S) = \min \left\{ \left[\sum_{c \epsilon J(e)} g_c(s) - d_k(s) \right]^+ , \delta_k\left(e, S'\right) \right\};$$

and finally, for every service $s \epsilon S \setminus S'$,

$$\varphi_k(e,s) = \min \left\{ \left[\sum_{c \epsilon J(e)} g_c(s) - d_k(s) \right]^+ , \delta_k(e,s) \right\}.$$

Although the above formulae may seem rather complicated, the computations required to obtain these estimates are really quite simple. Finally, note that these calculations implicitely define a loading of local demands on the equipements present at the node at that period.

4.3 Implementing the branching rules

The definition of a state, as given in section 4.1, implies that every equipment t can be represented by a binary variable $u(t)$, with $u(t) = 1$ if t present in that state (i.e., if $t \in J(e)$) and $u(t) = 0$ otherwise. Given a specific set of potential equipments J, any state e is then uniquely defined by the vector of binary variables reflecting the status of each equipment in J. However, with this definition it is not possible to distinguish between an equipment that has yet to be installed and one that has already been retrieved. This distinction is of paramount importance to the implementation of the branching rules in the DPs, since one of their roles is to force some equipments to be used with certainty in the NEP being generated.

In order to do so we have to enhance the information provided by the equipment status. Hence, to every equipment t of J affected by a branching constraint we will associate a *ternary* status variable $u(t)$ having the following interpretation:

$$u(t) = \begin{cases} 0, & \text{thas not yet been installed;} \\ 1, & t \text{ is present;} \\ 2, & t \text{ has been retrieved.} \end{cases}$$

This will allow us to guarantee that the chosen NEP effectively respects all branching constraints active at that point of the solution process.

Going back to the branching constraints defined in section 3.3, these have an impact both on the choice of the set of potential equipments J and on the dynamic program itself. The constraints resulting from rule 1 will force J to include or exclude some equipment types and those that come from rule 2 will force J to have at least (or less than) a certain number of equipments of a given type. However, this will only force J to respect these constraints. Since J is just the set of potential equipments from which provisioning at that node is to be performed, nothing yet ensures that the selected NEP will effectively use all equipements required by the branching constraints. To attain this objective, all terminal states e that do not respect these constraints will be declared "illegal" and their salvage value $V(e)$ will be set to infinity. This will force the constraints to be respected without imfluencing the remainder of the NEP.

In the case of the sequence of existence (rule 3), the role of the resulting constraints is to ensure that in the selected NEP some particular equipment be either present or absent at a given period (stage). Evidently, this implies first that the equipment in question be present in J. Then, to force the status of that equipment to reflect the constraint all states e that do not comply with it are declared "illegal" for the period in question and their local cost $C_k^\ell(e)$ are set to infinity.

As we just saw, all constraints originating from the branching rules can be implemented in the DP in a straightforward manner by acting on the selection of

J and on the local costs. This does not alter the DP structure in any way, thus preserving the efficiency of the solution procedure. We can therefore ensure that all the pertinent information provided by the B&B is effectively taken into account in the SPs and the combined B&B branch-and-bound/column-generation algorithm should thus converge.

5. HOMING AND LOADING PROCEDURE

Having a network evolution strategy that includes remote concentrators makes sense only if a feasible homing pattern can be found for them. Since in the SEM this aspect of the problem was relaxed we must now determine if the tentative solution it provided us with is really feasible. To do so we have to find a feasible homing pattern for all the remotes this proposed strategy contains, and since we are looking for the minimal cost evolution strategy we must try and find the cheapest feasible homing pattern possible.

However, as indicated earlier, most equipments are versatile with regards to the services they can accomodate. Consequently, loading local demands on a set of equipments at a given node can generally be made in several alternative ways. To each different loading correspond different residual switching capacities and thus different hosting capacities. Figure 3 illustrates this point by presenting two alternate loadings of local demands, only one of which is able to accomodate the outstanding homing demand, that is loading #2. If loading #1 were to be chosen, that node would not have the adequate residual switching capacity to host the external demand and the homing would not be possible. This having been said, it should be quite obvious

Capacity	Equipment type		
Service	Switch A	Switch B	Remote
1	1000	600	200
2	300	---	---
3	---	---	200
4	---	300	---
TOTAL	1000	600	200

Service demands		
Service	Volume	Type
1	1000	Local
2	200	**Homed**
3	200	Local
4	200	Local

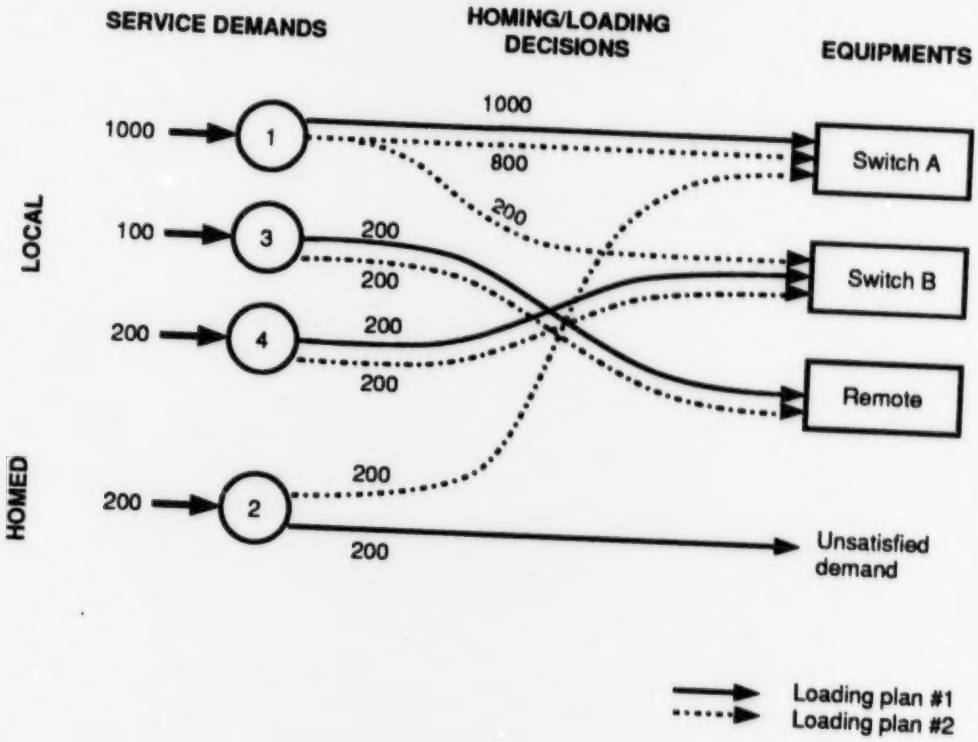

Figure 3: Example of alternate loadings

that the implicit loading that was performed in the SEM when generating the plans cannot be considered as final and used directly as the basis on which to make homing decisions. To maximize the chances of finding a feasible homing pattern for any strategy given by the SEM, both the homing and the loading decisions must be taken simultaneously. We must therefore reconsider all the loading decisions of the SEM. This is why we have to solve a *homing and loading sub-problem* and not just a homing sub-problem. Furthermore, reopening the loading problem will allow us to evaluate more precisely the additional switching capacity needed in the case where no feasible homing and loading strategy can be found than if we solved the homing problem alone (considering the SEM loading as fixed). This evaluation of additional needs is what serves to adjust the aggregate homing constraints in the SEM through the Homing Adjustment Loop and enables the overall procedure to iterate as described in section 3.1.

The problem we are faced with is a very complicated one because of the combinatorial nature of both the loading and homing decisions and also because of the type of capacity constraints that have to be respected. To this already grim picture, one has to add further complicating aspects like homing and re-homing policies: what is to be allowed and forbidden by the model to reflect actual or foreseen industry practices. Considering the level of complexity all of this implies and recalling the limited importance of this aspect of the solution with regards to its overall cost, solving the homing and loading sub-problem exactly would certainly not be a good investment of time or effort. Hence, the only valid approach in this

context is to resort to heuristic procedures to find a feasible homing and loading plan that is also "reasonably good" costwise. Several methods can be envisioned to carry out this task ranging from constructive heuristics that progressively build a homing and loading plan to exchange heuristics that iteratively improve an infeasible plan until it eventually becomes feasible (modeling the problem as a multicommodity flow problem, for instance).

The Homing and Loading procedure we propose can be described as a hierarchical constructive heuristic. It aims at progressively reducing the range of homing and loading alternatives for service demands by first identifying and implementing gradually all decisions for which there is no alternative (i.e. all *forced* decisions). The idea behind this way of constructing the homing and loading plan is that by taking these forced decisions first, many of the alternatives initially present will disappear, which might in turn render other decisions "forced" and so on, creating a sort of domino effect. When all forced decisions have been implemented, the procedure then uses a set of heuristic "rules of preference" to complete the homing and loading plan. We say that this procedure is hierarchical because the order in which the decisions are incorporated into the plan reflects their importance relative to one another and also because this order is based on notions of priorities as they apply to both services and equipments. For instance, services demanding highly specific equipments will be treated before those having a wide range of possible supporting equipments, and versatile equipments will generally be loaded after equipments with low service compatibilities.

Roughly speaking, the procedure uses 3 modules to address the four types of decisions that can be encountered. Listed by decreasing order of priority they are: Forced Loading module, Forced Homing module, and the Preferential Homing and Loading module which addresses both types of preferential decisions. The first decisions that have to be incorporated are naturally the forced loadings since they concern local demands which have a higher priority than homing demands and also since they need to be implemented in order to determine the available hosting capacities. After all forced loadings have been implemented, the procedure examines and executes all forced homings. If new forced loadings are created in the process, the procedure then returns to the Forced Loading module to treat them before resuming the forced homing phase. The same pattern of construction applies to the Preferential Homing and Loading decisions resulting in the structure illustrated in figure 4. All through the procedure, each time an infeasible loading or homing is encountered, the amount of demand that could not be satisfied is cumulated in a matrix, before being eliminated from further consideration (in essence that demand is temporarily considered as satisfied). The procedure is thus allowed to continue the loading process enabling it to "measure" the cumulative amount of capacity (per service and per period) that is "lacking" in the network to completely satisfy demands with the current switching evolution plan.

It is with these values that the Homing Adjustment Loop will adjust the right-hand side of constraints (9) to (11) of SEM(R). A complete description of the procedure can be found in Soriano (1987).

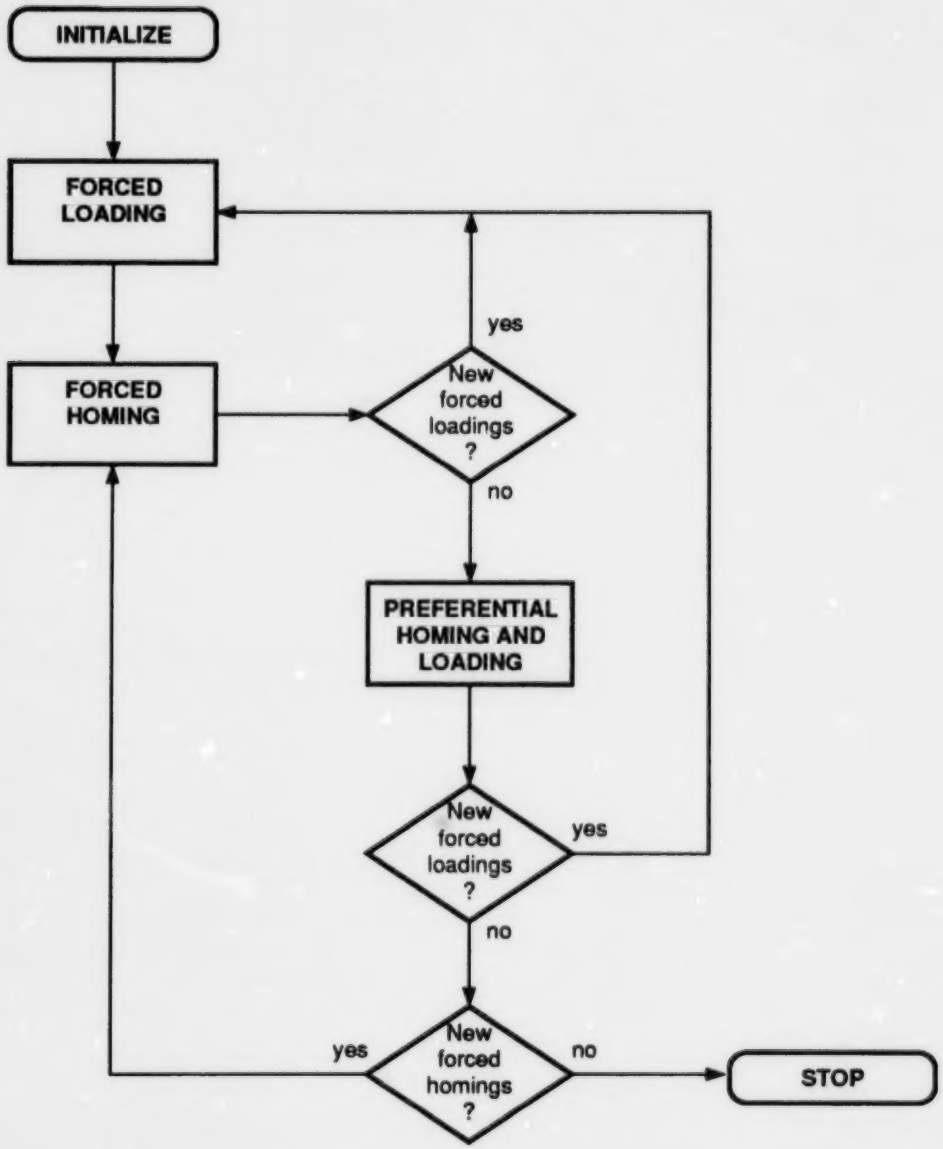

Figure 4: Structure of the Homing and Loading Procedure

6. A DECISION SUPPORT SYSTEM

As was mentionned in the introduction, the model we propose is conceived as the central element of a Decision Support System (DSS) rather than as a stand alone optimization model. This choice was motivated by the complexity of the problem at hand, but also by the diversity of non-quantifiable elements that need to be considered in a real-life setting. Indeed, it was indicated to us from the outset by several industry practitioners that what was really needed was a planning tool powerful and flexible enough to enable a planner to examine a given problem instance from several alternate viewpoints. Such a tool would provide a convenient way for evaluating and comparing policies, equipment development options, serving arrangements, an so on.

It is with this in mind that the overall modeling and solution approaches were developed, resulting in the system illustrated in figure 5.

The proposed DSS includes, in addition to the SEM, the Homing and Loading Procedure (HLP) and Homing Adjustment Loop (HAL) introducted in section 2, two new modules, the Filter and the Post-Optimizer. In this section we will briefly describe the role and nature of these modules. They constitute the interface between the user and the heart of the system, that is the SEM. Both of them are optional in a sense since they are not required by the SEM, however, it is with the flexibility and ease of use they provide that this DSS becomes a powerful planning tool.

The role of the Filter is twofold. It serves first of all to control and eventually reduce the size of the problems to be solved. As can be recalled, one of the main difficulties encountered in modeling the problem was the size of the resulting

mathematical programs. Even though a relatively concise formulation was obtained, it could still be useful to have means of reducing the size of the problems sent to the SEM. This can be achieved by eliminating from consideration all alternatives judged "unreasonable" and similarly imposing decisions judged highly probable. By taking these decisions in the Filter we can eliminate all associated variables from the problem, thus significantly reducing its size and consequently the time needed to solve it. The criteria used to eliminate an option or force a decision are specified to the Filter by the user.

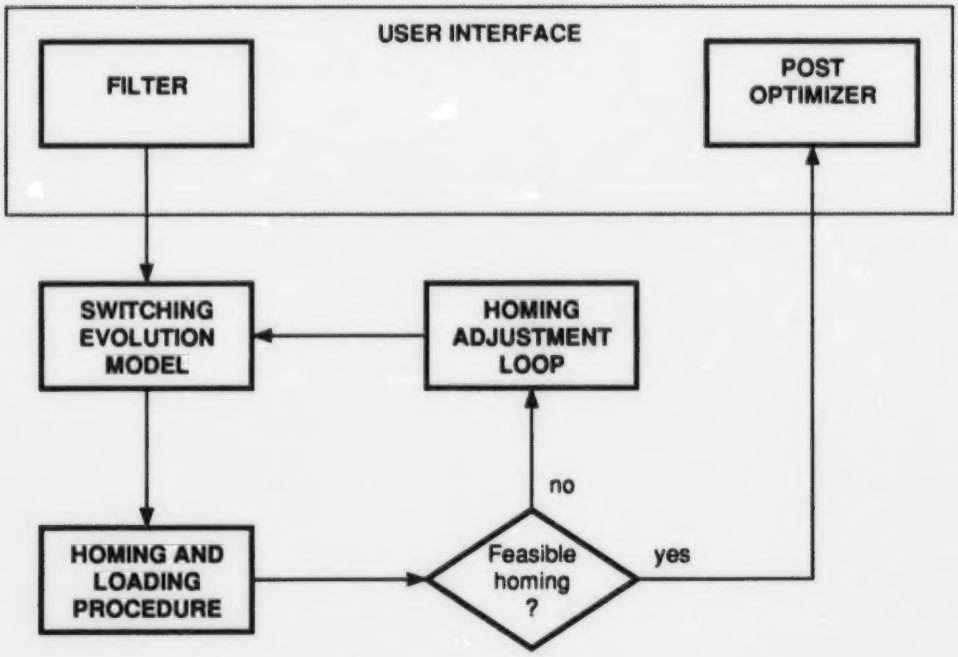

Figure 5: The decision support system

With this idea of fixing beforehand a certain number of elements the Filter can also be used by the planner to force the SEM to explore a particular dimension of a problem instance. Using it in this fashion, a planner will be able to define as many "versions" of the same problem as needed to represent alternate service options, management decisions or random events, that need to be studied. Solving these problems and then comparing the resulting Switching Evolution Plans will enable him/her to asses the impact of each alternative on the network evolution and to choose the one he/she feels is the best.

The Post-Optimizer module constitutes the last step in the solution process. A module of this type is necessary because the solution provided by the SEM and HLP may not be directly applicable. Indeed, the model being a "simplified" version of the real problem, it is probable that the solution it proposes contains some elements that are not consistent with company practices or that the planner may consider undesirable given his/her experience. Furthermore, since we are solving a relaxed version of the problem, the solutions we obtain are sub-optimal. In order to find the best solution possible, it would certainly be useful to have a Post-Optimizer powerful enough to allow the planner to explore a neighbourhood around the proposed solutions without having to solve the problem anew. This would enalbe the system to benefit from the planner's experience and knowledge of the problem.

7. CONCLUSION

We have addressed the problem of optimizing switching evolution in a multi-service environment when multiple equipment transitions are allowed at each node of

the network over the planning-horizon. This extremely difficult problem had never been, to our knowledge, studied as such in the past. We have proposed a modeling approach which focusses on the key decisions at hand, i.e. equipement choices. This, coupled with the use of sophisticated mathematical programming techniques such as combined branch-and-bound/column-generation schemes, has allowed us to come up with a model which captures the crucial economic trade-offs involved in the optimization of switching evolution, while remaining tractable for real-sized networks.

Finally, we have shown how this model can be integrated within a decision-support system to provide planners with a comprehensive, powerful and flexible tool to analyze and evaluate alternate switching evolution strategies.

Using these results as a starting point, it should now be possible for researchers to define customized switching evolution models addressing the particularities of specific networks and to carry out full-scale implementations of such models.

ACKNOWLEDGEMENT

The authors acknowledge the technical and financial support provided by Bell Northern Research at Ile-des-Soeurs, Montréal (QC), Canada, all through this research. This work was also partially funded by grants from N.S.E.R.C. and F.C.A.R. Finally, the authors also acknowledge the work of a former student, Khalid Idrissi-Kaïtouni, in connection with the implementation of the dynamic programming sub-problem and the heuristic selection procedure for the potential set of equipments.

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